NONLINEAR PHYSICS AND MECHANICS

MSC 2010: 93B11, 65P99, 65K10

Similarity and Analogousness in Dynamical Systems and Their Characteristic Features

S. Yu. Misyurin, G. V. Kreinin, N. Yu. Nosova

Mathematical models describing technically oriented dynamical systems are generally rather complex. Very time-consuming interactive procedures have to be used when selecting the structure and parameters of the system. Direct enumeration of options using such procedures can be avoided by applying a number of means, in particular, dimension methods and similarity theory. The use of dimension and similarity theory along with the general qualitative analysis of the system can serve as an effective theoretical research method. At the same time, these theories are simple. Using dimension and similarity theory, it is possible to draw conclusions when considering phenomena that depend on a large number of parameters, but so that some of them become insignificant in certain cases.

The combined method of using the theory of similarity, analogousness and methods developed by the authors for testing the drive model provides insight into its dynamics, controllability and other properties. The proposed approach is based on systematization and optimization

Received July 03, 2019

Accepted August 23, 2019

This work was supported by the Russian Foundation for Basic Research, project No. 18-29-10072 mk “Optimization of Nonlinear Dynamic Models of Robotic Drive Systems Taking into Account Forces of Resistance of Various Nature, Including Frictional Forces”.

Sergey Yu. Misyurin

ssmmrr@mail.ru

National Research Nuclear University MEPhI (Moscow Engineering Physics Institute),
Kashirskoe sh. 31, Moscow, 115409 Russia

German V. Kreinin

kreinin@mail.ru

Blagonravov Mechanical Engineering Research Institute of the Russian Academy of Sciences,
M. Kharitonyevskiy per. 4, Moscow, 101990 Russia

Natalia Yu. Nosova

natahys@mail.ru

National Research Nuclear University MEPhI (Moscow Engineering Physics Institute),
Kashirskoe sh. 31, Moscow, 115409 Russia
Blagonravov Mechanical Engineering Research Institute of the Russian Academy of Sciences,
M. Kharitonyevskiy per. 4, Moscow, 101990 Russia

of the process of forming a dimensionless model and similarity criteria, its focus on solving the formulated problem, as well as on special methods of modeling and processing of simulation results. It improves the efficiency of using similarity properties in solving analysis and synthesis problems. The advantage of this approach manifests itself in the ultimate simplification of the dimensionless model compared to the original model. The reduced (dimensionless) model is characterized by a high versatility and efficiency of finding the optimal and final solution in the selection of parameters of the real device, as it contains a significantly smaller number of parameters, which makes it convenient in solving problems of analysis and, in particular, synthesis of the system.

Dimension methods and similarity theory are successfully applied in the study of dynamical systems of different classes. The problems that arise are mainly related to the selection of a rational combination of the main units of measurement of physical quantities, the transition to dimensionless models and the formation of basic similarity criteria. The structure and the form of the dimensionless model depend on the adopted units of measurement of the variables appearing in the equations of the model and on the expressions assigned to its coefficients. Specified problems are solved by researchers, as a rule, by appealing to their intuition and experience. Meanwhile, there exist well-known systematized approaches to solving similar problems based on the method of the theory of analogousness.

Keywords: similarity, analogousness, hydraulic drive, dynamical system, dimensionless parameters

1. Introduction

Hydraulic drives play an important role in industry due to their characteristics such as high power capacity, operation speed, running smoothness, and positioning accuracy. The scope of application of hydraulic drives is diverse and includes material and fatigue testing machines, equipment for steel and aluminum industry, robotics etc. All these industries place fairly high demands on the performance of drives. Over the last two decades there has been a huge amount of research in this direction [1–6, 8–11].

But due to the complexity of the mathematical description of many technically oriented dynamical systems, rather labor-intensive interactive (dialogue) procedures are used in choosing their structure and parameters. Direct enumeration of options in using such procedures can be avoided by applying a number of means, in particular, dimension methods and similarity theory. These methods are based on using dimensionless complexes of physical parameters of the system (criteria of similarity and relation between them) and bringing the mathematical description of the system to dimensionless form [12–14]. As a result, additional possibilities are opened up to reveal general features of dynamical processes, which considerably facilitates making final decisions. So far there have been extremely few analogous studies.

As the experience shows, each specific problem concerning the dynamics of a mechanical system requires a special approach to forming the dimensionless model and similarity criteria. The structure and the form of the dimensionless model depend on the adopted units of measurement of variables appearing in the equations of the model, and on the expressions assigned to its coefficients. These factors are a priori unknown and are usually formed by researchers by appealing to their intuition and experience, which adds uncertainty to the process of transition to the dimensionless model and does not guarantee a high efficiency in its use.

The proposed approach to forming dimensionless models of dynamical systems enhances the efficiency of using similarity properties in solving the problems of analysis and synthesis. Firstly, it is based on the systematization and optimization of the process of formation of the dimensionless model and similarity criteria, and is aimed at solving the formulated problem. Secondly, this approach is based on special methods of modeling and processing of the modeling results. As for the former, of particular interest are methods of “analogousness theory” (according to the author’s terminology) [15]. The essence of these methods is that the units of measurement of variables and expressions of the parameters of the transformed model are found by solving a system of specially formed auxiliary equations. Further steps to solve the formulated problem depend on its characteristic features, complexity and, as a rule, involve jointly using specific similarity and analogousness methods. As an example, we consider the problem of synthesis of a conventional drive with a hydraulic motor (Fig. 1).

Fig. 1. Conventional drive with a hydraulic motor.

In recent years, there has been a large body of research devoted to the dynamics and control of systems with a hydraulic motor [16–20]. All these studies are characterized by a unified approach, which consists in formation and numerical modeling taking into account specific features of the mechanical (drive) system. In the case of such an approach it is difficult to understand the special features of the solution of the problem of parametric synthesis. The transition to dimensionless variables using the methods of “analogousness theory” results in a more universal and convenient mathematical model for solving synthesis problems.

To describe the dynamics of a hydraulic drive with inertial and power loads, we use the system of equations [1, 21–23] reduced to the output y -coordinate:

$$\begin{aligned} m\ddot{y} &= Fi\Delta p + P_L, \\ \Delta \dot{p} &= \frac{4E\dot{y}_{oh}}{l} \left(\gamma(1 - \text{sign}(\gamma)) \sqrt{\frac{\Delta p}{p_M}} - \frac{\dot{y}}{\dot{y}_{oh}} \right). \end{aligned} \quad (1.1)$$

This system includes the variables: the reduced piston movement y , the pressure difference on the piston, Δp , time t , as well as the parameters: load mass m , the active area of the piston, F , the transfer factor of the mechanism, i , constant power load P_L , the pressure in the power source, p_M , the coefficient of elasticity of the operating fluid, E , the average reduced length of the cavities of the cylinder’s fluid, l , and the reduced maximal steady speed of the piston, $\dot{y}_{oh}$. The control function γ is the opening β of the sectional area of channels, f , of the

distributor in the function of time and other variables:

$$A_1 \ddot{\lambda} = \Delta\sigma + \Delta\sigma_L, \quad (1.2)$$

$$\Delta\dot{\sigma} = A_3 \left(\gamma(1 - \text{sign}(\gamma)A_4\sqrt{\Delta\sigma}) \right) - A_2\dot{\lambda},$$

$$A_1 = \frac{mq_1}{q_2^2 q_3 F i}, \quad A_2 = \frac{q_1}{q_2 \dot{y}_{oh}}, \quad A_3 = \frac{4q_2 E \dot{y}_{oh}}{\lambda_s q_3 q_1}, \quad A_4 = \frac{q_3}{p_M}. \quad (1.3)$$

By the method of analogousness theory, equations (1.1) are transposed into dimensionless form by making a change of variables, i.e., by replacing the variables with their dimensionless analogs λ , τ , $\Delta\sigma$ according to the relations $y = q_1\lambda$, $t = q_2\tau$, $\Delta p = q_3\Delta\sigma$. The change of variables, the substitutions $l = iq_1\lambda_s$, $P_L = Fi\Delta p_L$ and simple transformations yield the transformed system (1.2) and the system of equations (1.3) describing the relations between the coefficients A_i of the system (1.2) and q_1, q_2, q_3 :

The system (1.3) includes four constraint equations $A_i(q_j)$, and A_i , like q_j , are unknown for the time being. This allows us to assign arbitrary values to three coefficients A_i of four, for example, to set $A_1 = A_2 = A_4 = 1$ and determine the values of q_j from these conditions. As a result, we obtain the system

$$A_1 = A_2 = A_4 = 1, \quad A_3 = \frac{4q_2 E \dot{y}_{oh}}{\lambda_s q_3 q_1}.$$

The solution of this system leads to

$$q_1 = \frac{m \dot{y}_{oh}^2}{p_M F i}, \quad q_2 = \frac{m \dot{y}_{oh}}{p_M F i}, \quad q_3 = p_M, \quad A_3 = K_S = \frac{4\varepsilon \dot{y}_{oh} q_2}{\lambda_s q_1}, \quad (1.4)$$

where $\dot{y}_{oh} = \frac{f}{Fi} \sqrt{\frac{p_M}{\rho}}$ is the density of the operating fluid and $\varepsilon = \frac{E}{p_M}$ is the dimensionless analog of the coefficient of elasticity of the fluid. Below we present a transformed model of the drive in the final form:

$$\ddot{\lambda} = \Delta\sigma + \Delta\sigma_L, \quad (1.5)$$

$$\Delta\dot{\sigma} = K_S \left(\gamma(1 - \text{sign}(\gamma)\sqrt{\Delta\sigma}) - \dot{\lambda} \right).$$

It follows from (1.5) that the equations contain the coefficient $K_S = A_3$, which serves as a stiffness (compliance) criterion of the dynamical system of the drive, and the power load parameter $\Delta\sigma_L$. It has turned out that the influence of K_S and $\Delta\sigma_L$ is insignificant when it comes to large ranges. All this appreciably enhances the efficiency of the use of computer modeling in solving problems of analysis and especially synthesis of drive systems. Also, this opens up prospects of generalizing simulation results, for example, in the form of tables adapted to the chosen laws γ of drive control.

The general scheme of the process of synthesis of a positioning drive that transfers an object from one position to another is illustrated below using the system (1.5) with the control law [12, 13]:

$$\gamma = \alpha_1(\lambda_e - \lambda) + \alpha_2(\dot{\lambda}_e - \dot{\lambda}), \quad (1.6)$$

where $\lambda_e = 0.5(1 - \cos(\omega\tau))$ is the chosen test law, which characterizes the smooth motion of the output link from point $\lambda = 0$ to point $\lambda_S = 1$, and the conditional frequency $\omega = \pi/\tau_S$

represents here the time of motion, τ_S . The ability of the power block to implement the given test law is estimated from the accuracy of control action (1.6), i.e., from the closeness of the test law and the reproducible law.

Figure 2 shows solutions obtained by modeling Eqs. (1.5) and (1.6) under the fixed initial conditions $\lambda_0 = 0$; $\dot{\lambda}_0 = 0$; $\Delta\sigma_0 = 1$. The final translation coordinate $\lambda_S = 1$ is also fixed. Thus, the course of the process is mainly determined by the parameter τ_S . This time should be set to be maximally small since, as τ_S decreases, the energy performance of the power block increases, and the latter copes faster with the task at hand. However, the limit of decrease of τ_S does exist, and is determined from the accuracy of implementation of the law (1.6) in the process of modeling. In this case, the value $\tau_{Smin} \approx 4$ was chosen as such a limit; it turned out to be possible to extend this value to the load turndown range within the limits $\Delta\sigma_L = 0 - 0.5$. For large loads the value should be chosen to be $\tau_{Smin} > 4$ and increased as $\Delta\sigma_L$ grows above 0.5.

Fig. 2

As for the stiffness (compliance) criterion of the dynamical system of the drive K_S , its value affects only the quality of the process: the curves in Figs. 3a and 3b have been obtained for $K_S = 200$ and $K_S = 300$, respectively. Comparison of these processes shows that, as K_S increases, the process improves. In the limiting case, when K_S reaches sufficiently large values, the system of equations reduces to one equation $\ddot{\lambda} = (1 - (\dot{\lambda}/\gamma)^2)\text{sign}(\gamma)$, and the oscillatory disturbances almost disappear (Fig. 3c).

In view of the above, the process of synthesis as such turns into a sequence of simple computational operations. Let, for example, the actual time of motion of mass, t_S , the stroke of the mass, y_S , and the technical parameters p_M , i be given. First, $q_1 = y_S/\lambda_S = y_S$, $q_2 = t_S/\tau_S$ and the scale velocity coefficient $q_{12} = q_1/q_2 = \dot{y}_{oh}$ are calculated. Next, the parameters $P_y = p_M Fi$, F and f are determined from the formulae presented above. For the hydraulic piston motor it is also necessary to take into account the speed limitation. The maximal speed value (at point $\tau = 2$) is $\dot{\lambda}_{max} \approx 0.4$. Consequently, the maximal speed of the piston will be $\dot{x}_{max} = \dot{y}_{max}i = 0.4q_{12}i = 0.4\dot{y}_{oh}i$. This speed value is compared with the limit established by technical requirements.

Fig. 3

2. Conclusion

In contrast to the classical similarity theory, the process of formation of the dimensionless model of the dynamical system and of the similarity criteria included in it obeys here certain rules which lead to the optimal set of basic units of measurement of physical quantities, which corresponds to the problem to be solved. In comparison with the initial model, the reduced (dimensionless) model is characterized by a high versatility, and contains a much smaller number of parameters, which makes it convenient in solving problems of analysis and, in particular, synthesis of the system. The process of synthesis in the example given above is carried out as a search for the limiting value of one or a small number of basic parameters of the reduced system, from the condition of qualitative tracking of the adopted test law. In the example considered in the paper, we have used a system of independent units of measurement which consists of speed, acceleration and pressure. As a result of reduction of the model, the number of parameters was diminished from 10 to 2.

In constructing the initial model, we have, for clarity, left out of account many factors typical of real systems, such as friction, other types of loads, dynamics of the control loop etc. All these additional factors can, as necessary, be incorporated into the model in the form of a test law or in some other way.

References

- [1] Merritt, H. E., *Hydraulic Control Systems*, New York: Wiley, 1991.
- [2] Has, Z. and Rahmat, M. F., Application of Self-Tuning Fuzzy PID Controller on Industrial Hydraulic Actuator Using System Identification Approach, *Int. J. Smart Sensing Intell. Syst.*, 2009, vol. 2, no. 2, pp. 246–261.
- [3] Huang, Y. J., Kuo, T. C., and Lee, H. K., Fuzzy-PD Controller Design with Stability Equations for Electro-Hydraulic Servo Systems, in *ICCAS'2007: Proc. of the Internat. Conf. on Control, Automation and Systems (Seoul, Korea, Oct 2007)*, pp. 2407–2410.
- [4] Alleyne, A. and Liu, R., A Simplified Approach to Force Control for Electro-Hydraulic Systems, *Control Eng. Pract.*, 2000, vol. 8, no. 12, pp. 1347–1356.
- [5] Lizarde, C., Loukianov, A. G., and Sanchez, E., Force Tracking Neural Block Control for an Electro-Hydraulic Actuator via Second-Order Sliding Mode, *Int. J. Robust Nonlinear Control*, 2008, vol. 18, no. 3, pp. 319–332.
- [6] Avila, M. A., Loukianov, A. G., and Sanchez, E. N., Electro-Hydraulic Actuator Trajectory Tracking, in *Proc. of the American Control Conference (Boston, Mass., 2004): Vol. 3*, pp. 2603–2608.
- [7] Cotsaftis, M. and Keskinen, E., Smooth High Precision Contact Position Control of Rotating Cylinders with Hydraulic Actuators, in *Proc. of the 12th IFToMM World Congress (Besancon, France, 2007)*, pp. 738–743.
- [8] Lu, X., Du, F., Jia, Q., Ren, B., and Wang, X., Sliding Mode Force Control of an Electrohydraulic Servo System with RBF Neural Network Compensation, *Mechanika*, 2019, vol. 25, no. 1, pp. 32–37.
- [9] Guan, C. and Pan, S., Adaptive Sliding Mode Control of Electro-Hydraulic System with Nonlinear Unknown Parameters, *Control Eng. Pract.*, 2008, vol. 16, no. 11, pp. 1275–1284.
- [10] Raade, J. W. and Kazerooni, H., Analysis and Design of a Novel Hydraulic Power Source for Mobile Robots, *IEEE Trans. Autom. Sci. Eng.*, 2005, vol. 2, no. 3, pp. 226–232.
- [11] Wang, X., Modeling and Control of a Torque Load System with Servo Actuators Dynamics, in *Proc. Inst. Mech. Eng. G J. Aer.*, 2016, vol. 231, no. 9, pp. 1676–1685.
- [12] Sedov, L. I., *Similarity and Dimensional Methods in Mechanics*, 10th ed., Boca Raton, Fla.: CRC, 1993.
- [13] Bukhgolts, N. N., *Basic Course of Theoretical Mechanics: P. 2*, Moscow: Nauka, 1972 (Russian).
- [14] Sonin, A. A., *The Physical Basis of Dimensional Analysis*, 2nd ed., Cambridge, Mass.: MIT, 2001.
- [15] Mamontov, M. A., *Similarity*, Moscow: Min. Oboron. SSSR, 1971 (Russian).
- [16] Yang, G., Yao, J., Le, G., and Ma, D., Adaptive Integral Robust Control of Hydraulic Systems with Asymptotic Tracking, *Mechatronics*, 2016, vol. 40, pp. 78–86.
- [17] Guo, K., Wei, J., Fang, J., Feng, R., and Wang, X., Position Tracking Control of Electro-Hydraulic Single-Rod Actuator Based on an Extended Disturbance Observer, *Mechatronics*, 2015, vol. 27, pp. 47–56.
- [18] Tri, M. N., Nam, C. N. D., Park, G. H., and Ahn, K. K., Trajectory Control of an Electro Hydraulic Actuator Using an Iterative Backstepping Control Scheme, *Mechatronics*, 2015, vol. 29, pp. 96–102.
- [19] Li, L., Huang, H., Zhao, F., Triebe, M. J., and Liu, Z., Analysis of a Novel Energy-Efficient System with Double-Actuator for Hydraulic Press, *Mechatronics*, 2017, vol. 47, pp. 77–87.
- [20] Márton, L., Fodor, S., and Sepehri, N., A Practical Method for Friction Identification in Hydraulic Actuators, *Mechatronics*, 2011, vol. 21, no. 1, pp. 350–356.

- [21] Misyurin, S. Yu. and Kreinin, G. V., Power Optimization Criteria of a Mechanical Unit of an Automated Actuator, *Dokl. Phys.*, 2015, vol. 60, no. 1, pp. 15–18; see also: *Dokl. Akad. Nauk*, 2015, vol. 460, no. 1, pp. 39–42.
- [22] Misyurin, S. Yu. and Kreinin, G. V., Dynamics and Design of a Power Unit with a Hydraulic Piston Actuator, *Dokl. Phys.*, 2016, vol. 61, no. 7, pp. 354–359; see also: *Dokl. Akad. Nauk*, 2016, Vol. 469, No. 3, pp. 302–307.
- [23] Cotsaftis, M. and Keskinen, E., Smooth High Precision Contact Position Control of Rotating Cylinders with Hydraulic Actuators, in *Proc. of the 12th IFToMM World Congress (Besancon, France, 2007)*, pp. 738–743.