NONLINEAR PHYSICS AND MECHANICS

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Estimation of Instabilities under the Joint Action of Laser Radiation and a Magnetic Field on a Plasma

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One of the main obstacles to the uniform laser compression of a fusion target is the plasma formation instability (the Rayleigh–Taylor instability is the most dangerous). In all the schemes considered, the impulsive character is important. In this case, not all possible plasma instabilities are dangerous, but only those that most rapidly increase with time (for example, Rayleigh–Taylor instability).

Keywords: laser, magnetic field, mathematical model, plasma target

1. Introduction

Recall the formation mechanism of the Rayleigh–Taylor instability: if acceleration of dense medium (target wall) is considered using a less dense (rarefied) medium (the acceleration vector is directed from the rarefied medium to the dense), at the contact boundary, disturbances grow exponentially, which are transformed over time into a system of nonlinearly interacting jets (Fig. 1). The presence of such instability in a compressible target plasma leads to a decrease in the degree of target compression and a decrease in the efficiency of energy transfer from the laser to the target plasma. Rarefied medium can be a massless magnetic field or laser radiation quanta.

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Fig. 1. Rayleigh–Taylor instability development scheme.

Let us estimate the influence of magnetohydrodynamic instabilities of the Rayleigh–Taylor, Richtmyer–Meshkov type on the compression process of the target, using separate results of [1–14].

2. Analysis of instabilities

Several factors (which can lead to the development of instability of the Rayleigh–Taylor type) cause the acceleration of the contact boundary. The first factor leading to the acceleration of the contact boundary is associated with a strong shock wave, which falls and then passes through the considered multi-layered cylindrical target, and this motion by the shock wave can lead to a noticeable compression of continuous media. The next factor causing the acceleration of the contact boundary can be a powerful instantaneous energy release (Joule heat) in the region of the layers (shells) of the target or the environment. It is obvious that, overall, the generation of the instability of the contact boundary in the target can be caused by the action of a combination of the first two methods of acceleration on it.

There are several ways of formulating the mathematical description of the target deformation process: in the first case, the target is considered as an elastic body, in the second case, as an elastoplastic body, in the third case, as a viscous magnetoplasma-dynamic medium. In this paper, we use the third method of formulating a system of equations that describe the process of target deformation and the instability development at the contact boundary. This formulation is based on the equations of magnetic hydrodynamics. However, this approach is approximate due to the fact that the loads experienced by the target wall significantly exceed its yield strength. Therefore, in principle, it would be optimal to model the deformation of the target by an elastoplastic body with a real deformation curve (for example, obtained as a result of experiments).

We will consider small perturbations of the hydrodynamic parameters of the plasma near the contact surface as the flow of an ideal fluid with density ρ , potential-dependent velocity $V = -\nabla\varphi$, (φ is the potential) and magnetohydrodynamic phase velocity (velocity of a fast magnetosonic wave $\sqrt{a} = \sqrt{c^2 + \frac{H^2}{4\pi\rho}}$, where $c^2 = \left(\frac{\partial P}{\partial \rho}\right)_s$ is the gas-dynamic sound velocity). The solution of such a problem is formulated as a solution of the wave magnetohydrodynamic equation $\frac{1}{a} \frac{\partial^2 \varphi}{\partial t^2} + \Delta \varphi = 0$ (in the one-dimensional version $\frac{1}{a} \frac{\partial^2 \varphi}{\partial t^2} + \frac{\partial^2 \varphi}{\partial n^2} = 0$, where n is the normal to the contact boundary), with a boundary condition of the form: $\left. \frac{\partial P}{\partial t} \right|_{CB} = 0$. We will seek a solution to these equations in the form of a plane wave $\exp(ikn)$. Then the dispersion relation has the form $\omega^2 = \pm ka$, and the solution is determined by the expression $\varphi \approx f(t) \exp(\pm ikn)$.

It follows that the displacement $\Delta(t)$ of the contact surface (in the equilibrium situation, it is at rest $\Delta = 0$), which is in the field of bulk forces (with acceleration a), can be described by

the relation:

$$\Delta = \Delta_0 \operatorname{ch}(\omega t) \cos(kr), \quad \Delta(t=0) = \Delta_0 \cos(kr), \quad (2.1)$$

where k is the wave number.

The oscillation frequency ω is found using the dispersion relation:

$$\omega^2 = \pm ak. \quad (2.2)$$

Thus, an increase in the value of the magnetic intensity will lead to a shift in the spectrum of oscillations in the high-frequency region.

The sign (+) in expression (2.2) corresponds to the case when the gradient vector of the normal (to the equilibrium form of the boundary) magnetic field components ∇B and the acceleration vector $\vec{a}$ of the contact boundary are directed in the same direction and $\operatorname{Re}(\omega) \neq 0$, $\operatorname{Im}(\omega) = 0$. In this case, Rayleigh–Taylor type instability develops.

The sign (−) corresponds to the multidirectional case of the vectors ∇B and $\vec{a}$ and $\operatorname{Re}(\omega) = 0$, $\operatorname{Im}(\omega) \neq 0$. In this case, the instability of Rayleigh–Taylor type is suppressed over time.

Thus, it follows from the above that it is possible to stabilize the position of the contact boundary by acting on it in a special way created by the magnetic field.

Let us estimate the value of azimuthal instabilities of the Rayleigh–Taylor type in the combined effect of laser radiation and a magnetic field on the plasma. Let us turn to the coordinate system associated with the spatially averaged position of the contact boundary. In this case, the volume forces acting on the contact boundary will either accelerate its movement (at the first two stages: $d^2r/dt^2 = a > 0$), or slow down (at the third stage: $a < 0$). The speed of azimuthal disturbances development in time is determined by the relation:

$$\omega^2 \approx \left| \frac{dV}{dt} \right| k \approx \left| \frac{dV}{dt} \right| \frac{\ell}{r}, \quad \frac{d\vec{V}}{dt} = \frac{d^2\vec{r}}{dt^2}, \quad (2.3)$$

where the wave number k is determined by the condition $\lambda\ell = 2\pi r$, $\ell = 1, 2, 3, \dots$; $\lambda = \frac{2\pi}{k}$ is the length of the azimuth wave. On the contact boundary between the solid wall of the target and the environment, the following boundary condition can be set:

$$k_m \frac{\partial T_s}{\partial r} = q - D\rho_0\Omega. \quad (2.4)$$

Assuming that the relationship is satisfied $\left| k_m \frac{\partial T_s}{\partial r} \right| \sim (|q|, D\rho_0\Omega)$, it is possible to determine the speed of movement D and acceleration dD/dt of the contact boundary in a relative coordinate system:

$$D = \frac{q(t)}{\rho_0\Omega}, \quad a = \frac{dD}{dt} = \frac{dq/dt}{\rho_0\Omega}, \quad (2.5)$$

where ρ_0 is the density of the target substance; Ω is the specific heat of evaporation (phase transition) of the target substance, $q(t)$ is the flux density of laser (or broadband) radiation incident on the target.

The magnitude of the acceleration of the contact boundary can be estimated using an approximate ratio of the form:

$$\frac{dV}{dt} = \frac{dD}{dt} + \frac{\frac{P}{\rho_0} \pm \frac{H^2}{2\rho_0\pi} - V^2/2}{\delta},$$

where δ is the target wall thickness; $\rho_c = \frac{m_0 m_e}{4\pi e^2} \left(\frac{2\pi c}{\lambda} \right)^2 = 1.83 \times 10^{-3} \frac{A\lambda}{z} \left(\frac{\text{g}}{\text{cm}^3} \right)$ is the critical plasma density; m_0 is the mass of one particle of a substance (molecule, atom or ion); $A \left(\frac{\text{g}}{\text{mol}} \right)$ is the atomic weight of plasma nuclei, λ (μm) is the laser radiation wavelength, z is the average ion charge; $P \approx q^{2/3} \rho_c^{1/3}$ is the value of the maximum pressure that can be achieved in the plasma.

Then the maximum time $1/\omega$ for the development of the Rayleigh–Taylor instability is

$$\frac{1}{\omega} \approx \sqrt{\frac{r}{|d\vec{V}/dt|}} \ell. \quad (2.6)$$

From relation (2.6) it follows that in the process of compression ($r \rightarrow 0$), the probability of instability increases. It also follows that an increase in the rate of heating of the target ($dq/dt \rightarrow \infty$) can have a negative role on the development of instability. However, by the time instant t_1 target material vapors form, near the contact boundary, a very dense layer of vapor that does not pass laser radiation $q(t)$ through it. The screening process is also facilitated by the compression of the plasma vapor layer using an external magnetic field. Here $t_1 = 0.63 q^{2/3} / \beta \rho_c^{8/3}$ is the moment of time from the beginning of the fusion target irradiation, starting from which the target begins to be shielded from laser radiation, ρ_c is the critical plasma density. In addition, a decrease in the target radius $r \rightarrow 0$ leads to a strong increase $\left(H(t) \sim \frac{1}{r^2(t)} \right)$ in the magnetic field “frozen” into the plasma (to an increase $\frac{H^2}{2\rho_0\pi}$) and to a decrease in the acceleration $\left| \frac{d\vec{V}}{dt} \right|$ of the contact boundary.

3. Impact of the external magnetic field on thermophysical and transport properties of substances

The degree of influence of the magnetic field on the transport coefficients of plasma (electrical conductivity, thermal conductivity) depends on the ratio of the collision frequency of electrons ν_e to the Larmor frequency $\omega_e = e \cdot B / m_e \cdot c = 1.76 \cdot 10^7 \cdot B$ of electron rotation in the magnetic field, where e is the electron charge, B is the magnetic induction in Gs, m_e is the electron mass, and c is the light speed.

In plasma, the magnetic field will exert an important influence on transport properties if $\nu_e / \omega_e \ll 1$. Given that the collision frequency of electrons in a fully ionized plasma can be determined by the formula

$$\nu_e = \frac{4\sqrt{2}\pi e^4 Z^2 n_i \Lambda}{3\sqrt{m_e} T_e^{3/2}} = 2.85 \cdot 10^{-5} \frac{\Lambda}{10} \cdot \frac{Z n_i}{T_e^{3/2}},$$

where Z is the ion charge, Λ is the Coulomb logarithm, T_e is the electron temperature in eV, n is the plasma density in cm^{-3} , we obtain the condition of the strong influence of the magnetic field on the plasma properties in the form

$$B \gg 1.6 \cdot 10^{-12} \frac{\Lambda}{10} \frac{Z n}{T_e^{3/2}}.$$

For gold plasma ($Z = 79$) with parameters $T_e \sim 1$ keV, $n \sim 10^{19}$ cm $^{-3}$ (which corresponds to the plasma density $\rho \sim 0.01$ g/cm 3 which is the target crown density in the insertional fusion) we have the following assessment ($\Lambda \approx 15$):

$$B \gg 1.6 \cdot 10^{-12} \frac{15 \cdot 79 \cdot 10^{19}}{10 (10^3)^{3/2}} \Rightarrow B \gg 5.99 \cdot 10^4.$$

In magneto-inertial fusion [20, 21] the magnetic fields were experimentally obtained $B_{\text{fus}} \sim 10^7$ Gs and the higher values can be achieved. Therefore, it is necessary to take into account the influence of superstrong magnetic fields on the transport properties of plasma.

The magnetic field, which affects the orientation of the spins of electrons or atoms in a gas having temperature T , is determined by the condition

$$\mu B = \frac{e\hbar}{2m_e c} B \gg k_B T \quad \text{or} \quad B \gg 1.49 \cdot 10^4 T \text{ [K]},$$

where μ is the Bohr magneton.

In our case the characteristic temperature is $T_{\text{char}} \sim 1$ keV $\approx 10^7$ K. Then

$$B \gg 1.49 \cdot 10^4 \cdot 10^7 \approx 10^{11},$$

which is larger than the values B achieved in the fusion. Thus, we assume that the magnetic field does not affect the orientation of the spins of electrons or atoms in the gas.

The magnetic field $B \sim 10^9$ Gs, in which the magnetic moment energy is larger than the characteristic binding energy of an atom or molecule (like $Ry = m_e e^4 / 2\hbar^2$), significantly affects the structure of atoms and molecules and greatly changes their binding and ionization energy. The fields under consideration are significantly smaller than $B \sim 10^{13}$ Gs, therefore we can neglect relativistic effects. Dependences of pressure, internal energy and entropy are given in Table 1. The previous evaluations have been confirmed. The field $B < 10^{10}$ Gs has almost no effect on the internal energy, pressure and entropy of the plasma.

Table 1. The dependence of the pressure, internal energy and entropy of the gold plasma on the value of the external magnetic field

$T = 1$ keV, $\rho = 0.01$ g/cm 3	$B = 0$ Gs	$B = 1$ Gs	$B = 10^6$ Gs	$B = 10^8$ Gs	$B = 10^{10}$ Gs	$B = 10^{12}$ Gs
P , Mbar	3.859	3.859	3.859	3.859	3.859	3.859
$U/N_i kT$	$1.167 \cdot 10^2$	$1.167 \cdot 10^2$	$1.167 \cdot 10^2$	$1.167 \cdot 10^2$	$1.167 \cdot 10^2$	37.5
$S/N_i k$	$1.125 \cdot 10^3$	$1.125 \cdot 10^3$	$1.125 \cdot 10^3$	$1.125 \cdot 10^3$	$1.125 \cdot 10^3$	$1.184 \cdot 10^3$

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