NONLINEAR PHYSICS AND MECHANICS

MSC 2010: 76F10, 76F20, 76E30, 76F25

Dynamical Model for the Anomalous Transport of a Passive Scalar in a Reverse Barotropic Jet Flow

V. P. Reutov, G. V. Rybushkina

The anomalous transport of a passive scalar at the excitation of immovable chains of wave structures with closed streamlines in a barotropic reverse jet flow is studied. The analysis is performed for a plane-parallel flow in a channel between rigid walls in the presence of the beta effect and external friction. Periodic boundary conditions are set along the channel, while nonpercolation and sticking conditions are adopted on the channel walls. The equations of a barotropic (quasi-two-dimensional) flow are solved numerically using a pseudospectral method. A reverse jet with a “two-hump” asymmetric velocity profile facilitating the faster transition to the complex dynamics of the Eulerian flow fields is considered. Unlike the most developed kinematic models of anomalous transport, the basic chain of structures becomes unsteady due to the birth of supplementary perturbations at saturation of barotropic instability. A regular (multiharmonic) regime of wave generation is shown to appear due to the excitation of a new flow mode. Immovable structure chains giving rise to anomalous transport are obtained in the multiharmonic and chaotic regimes. The velocity of the chains of structures was determined by watching movies made according to the computations of the streamlines. It is revealed that the onset of anomalous transport in a regular regime is possible at essentially lower supercriticality compared to the chaotic regime. Trajectories of the tracer particles containing alternations of long flights and oscillations are drawn in the chaotic regime. The time dependences of the averaged (over ensemble) displacement of the tracer particles and its variance are obtained for two basic regimes of generation with immovable chains of structures, and the corresponding exponents of the power laws are determined. Normal advection is revealed in the regular regime, while anomalous diffusion arises in both regimes and may be classified as a “superdiffusion”.

Keywords: barotropic reverse jet flow, chains of wave structures, dynamical chaos, anomalous advection and diffusion

Received July 22, 2019
Accepted August 15, 2019

This research was supported by the Russian Foundation for Basic Research (project No. 17-05-00747). The numerical simulation was funded by the Ministry of Science and Higher Education of the Russian Federation within the State task for the Institute of Applied Physics of RAS (project No. 0035-2014-0007).

Vladimir P. Reutov
reutov@appl.sci-nnov.ru
Galina V. Rybushkina
ryb@appl.sci-nnov.ru
Institute of Applied Physics, RAS
ul. Ulyanova 46, Nizhny Novgorod, 603950 Russia

1. Introduction

Zonal flows in the atmosphere and ocean of the Earth and other planets have complex velocity profiles with sections of a reverse flow (e.g., [1, 2]). Anomalous transport of a passive scalar in zonal flows with a reverse velocity profile is of great interest due to the phenomenon of atmospheric blocking responsible for the long-lived weather anomalies [3, 4]. In this work we are concerned with the anomalous transport of a passive scalar in a dynamical model of a reverse jet flow producing immovable chains of wave structures with closed streamlines. To date kinematic models of anomalous transport characterized by an arbitrary (to a considerable extent) choice of the wave field have been mainly elaborated. A classical two-wave kinematic model contains a basic wave forming a chain of structures with closed streamlines and a supplementary wave that ensures formation of stochastic layers [5, 6]. Within the dynamically consistent models supplementary waves are born due to the nonlinear evolution of the flow itself [7–12]. In the present work the appearance of supplementary wave perturbations is associated with the saturation of barotropic instability and the transition to dynamical chaos with increasing flow supercriticality. A jet flow with an asymmetric “two-hump” reverse velocity profile characterized by a faster transition to chaos compared to a conventional jet is considered. The velocity of chains of the wave structures in a reverse jet may be equal to zero or be very small. These immovable flow anomalies are analyzed in the present work that may be regarded as an extension of our previous studies [11, 12] devoted to the dynamical models of anomalous transport of a passive scalar in a shear layer and in jet flows with complicated velocity profiles. Therefore, we restrict ourselves to referencing to the relevant literature and description of a numerical procedure for the barotropic (quasi-two-dimensional) flows given in those works.

2. Basic equations

Let us choose a velocity scale equal to the maximum jet flow velocity U and introduce the spatial scale of the transverse velocity shift L . The equations of a barotropic (quasi-two-dimensional) flow on the beta-plane are written in dimensionless form in terms of absolute vorticity and streamfunction [10, 13]:

$$\frac{\partial \zeta}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial \zeta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \zeta}{\partial y} - \beta \frac{\partial \psi}{\partial x} = -\lambda (\zeta - \zeta_0) + R^{-1} \Delta (\zeta - \zeta_0), \quad \Delta \psi = -\zeta, \quad (2.1)$$

where x and y are the axes of the Cartesian coordinate system directed along and across the flow (in the geophysical flow they have the east and north directions, respectively), $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the two-dimensional Laplacian, ψ is the streamfunction related to the components of the horizontal velocity $\mathbf{v} = (u, v)$ by the expressions $u = \partial \psi / \partial y$ and $v = -\partial \psi / \partial x$; $\zeta = (\text{rot } \mathbf{v})_z = \partial v / \partial x - \partial u / \partial y$ is the vertical component of absolute vorticity, $R = UL/\nu$ is the Reynolds number defined by effective kinematic viscosity ν ; $\lambda = \lambda^{**}L/U$ is the dimensionless coefficient of external friction λ^{**} ; $\beta = \beta^{**}L^2/U$ is the dimensionless gradient of the Coriolis parameter β^{**} ; $\zeta_0 = -du_0/dy$ is the vorticity of the equilibrium plane-parallel flow with normalized velocity profile $u_0(y)$ maintained by an external force modeled by the inhomogeneous terms in the first equation of (2.1).

We consider a jet flow in a channel with rigid lateral walls $y = \pm y_1$ ($y_1 > 0$ is the half-width of the channel) specifying on the walls the following nonpercolation and sticking conditions:

$$\left. \frac{\partial \psi}{\partial x} \right|_{y=\pm y_1} = 0, \quad \left. \frac{\partial \psi}{\partial y} \right|_{y=\pm y_1} = 0. \quad (2.2)$$

The periodic boundary conditions are set at the boundaries of the computational domain in the longitudinal direction $x = 0, \Lambda$. In this case, the solution of the boundary problem (2.1) and (2.2) can be written in the form

$$\zeta = \bar{\zeta}(y, t) + \tilde{\zeta}, \quad \psi = \int \bar{u}(y, t) dy + \tilde{\psi}, \quad (2.3)$$

where $\tilde{\zeta}$ and $\tilde{\psi}$ are the oscillating (with respect to x) components of vorticity and streamfunction with period Λ , $\bar{\zeta}(y, t)$ and $\bar{u}(y, t)$ are the mean vorticity and velocity profiles. The substitution of (2.3) into (2.1) and (2.2) leads to simultaneous equations for the quantities $\tilde{\zeta}$, $\tilde{\psi}$ and mean velocity $\bar{u}$ given in [12, 14].

The problem is solved numerically for the reverse jet flow in the channel with half-width $y_1 = 4$, the velocity profile of which is approximated by the expression

$$u_0(y) = \frac{rf(y) - \mu}{1 - \mu} \cdot \frac{rf(y) - b}{1 - b}, \quad (2.4)$$

where $f = \text{sech}^2[e_2(y - \Delta y)] + g_1 \text{sech}^2[e_1(y + \Delta y)]$ is the basic “two-hump” velocity profile satisfying the condition $f \rightarrow 0$ as $y \rightarrow \pm\infty$, $e_2 = e_1 + \delta$ and b is the reverse parameter allowing the contribution of reverse flow to be controlled in the entire jet drain. The parameter δ is initially chosen to meet the condition $f(-y_1) = f(y_1)$ at fixed e_1 , g_1 , and Δy , after that $r = 1/\max f(y)$ and $\mu = f(\pm y_1)$ are calculated. Below, the velocity profile (2.4) with the parameters $g_1 = 0.9$, $e_1 = 1$, $\Delta y = 0.85$, $r = 0.888194$, $\mu = 0.00603453$, and $\delta = 0.017095$ is used. In this case, regardless of the magnitude of b , $u_0(\pm y_1) = 0$ to a high accuracy, and the maximum value of u_0 across the channel is equal to unity. The velocity profile (2.4) plotted at $b = 0.747$ is shown in Fig. 1 by the curve 1.

Fig. 1. Normalized profiles of the longitudinal velocity of a barotropic reverse jet in a horizontal channel between the walls $y = \pm 4$: 1 and 2 correspond to the velocity profiles of the primary (equilibrium) flow $u_0(y)$ and in the regime of dynamical chaos with an immovable basic chain of structures at $\text{Re} = 60$, $\beta = 0.4$, $b = 0.747$, and $\lambda = 0.09286$.

3. About the numerical scheme

The nonlinear system of equations (2.1)–(2.3) was solved numerically using a pseudospectral method in the treatment of the works [15, 16]. The quantities oscillating in x were represented in the form of a truncated complex Fourier series:

$$(\tilde{\psi}, \tilde{\zeta}) = \sum_{\substack{m=-K \\ m \neq 0}}^K (\hat{\psi}_m(y, t), \hat{\zeta}_m(y, t)) e^{ik_m x}, \quad (3.1)$$

where $k_m = mk_1$ ($k_1 = 2\pi/\Lambda$ is the wavenumber of the fundamental harmonics); $\hat{\zeta}_{-m} = \hat{\zeta}_m^*$ and $\hat{\psi}_{-m} = \hat{\psi}_m^*$ are the complex amplitude profiles of the spatial harmonics of vorticity and streamfunction, and K is the number of “nonzero” harmonics. A system of equations governing the complex profiles $\hat{\psi}_m, \hat{\zeta}_m$ simultaneously with the mean velocity profile $\bar{u}$ is given in [11]. The derivatives with respect to y were approximated by finite differences to an accuracy of the second-order terms. The number of complex harmonics in the discrete Fourier transform was $N = 64$, and the number of discretization nodes in y was adopted to be equal to 200. For excluding the aliasing phenomenon, the value of K was chosen from the condition $K \simeq N/3$ [15, 16]. A large system of ordinary differential equations for unknown variables $\hat{\zeta}_m$ and $\bar{u}$ taken at the discretization nodes was solved using the Runge-Kutta method. The relationship between $\hat{\zeta}_m$ and $\hat{\psi}_m$ at the nodes was resolved using difference sweeping. The initial profiles of $\hat{\zeta}_m$ were determined through some identical (for all harmonics) initial profiles of $\hat{\psi}_m$, which had small amplitude factors with pseudorandom phases and met the boundary conditions (2.2).

4. The transition to the dynamical chaos

The results of computations of the neutral curves $\lambda(k)$ and the corresponding phase velocity $c(k)$ of the bending mode of the reverse jet flow with the velocity profile (2.4) are displayed in Fig. 2. The well-known relation between the eigenvalues of the linear problem and the external friction coefficient was taken into account [13]: $\omega(k; \beta, R, \lambda) = \omega(k; \beta, R, 0) - i\lambda$, where k is the wavenumber and ω is the wave frequency. This formula gives rise to the expressions $\lambda(k) = -\text{Im} \omega(k; \beta, R, 0)$ and $c(k) = \text{Re} \omega(k; \beta, R, 0)/k$. The critical value of $\lambda_c = \max \lambda(k)$ was reached at the critical wavenumber $k = k_c$. When solving the nonlinear problem, we took $k_1 = k_c/7$ in the expansion (3.1), thereby, after loss of stability (at $\gamma > 1$), a stationary chain containing 7 wave structures with closed streamlines appeared in the computational domain. An increase in the supercriticality $\gamma = \lambda_c/\lambda$ was attained by a jump-like decrease of λ step by step at fixed $R = 60$ and $\beta = 0.4$ (in the real flow this corresponds to a decreasing external friction coefficient). A temporal interval for the sought solution after each jump of λ was chosen to obtain an established regime of generation for all harmonics. To specify the initial conditions after each jump in λ , the solution obtained before this jump was used. Besides, small perturbations were added to model the noise of the real flow (see [11, 14] for details). For determining the drift velocity of the chain of structures, a movie showing the streamfunction evolution was first recorded in the reference frame moving with trial velocity c , and then the value of c , at which the chain became immovable, was sought [12].

Figure 3 shows the temporal evolution of the integral amplitudes of spatial harmonics of the longitudinal velocity $B_j = \left(\int_{-y_1}^{y_1} |\hat{u}_j|^2 dy \right)^{1/2}$, where j is the harmonic number and

Fig. 2. Neutral curves $\lambda(k)$ and phase velocity $c(k)$ of the reverse jet bending mode. Curves 1, 2, 3 show $\lambda(k)$ and 1', 2', 3' give $c(k)$ at $b = 0.787, 0.775$, and 0.747 , respectively. Hereinafter the graphs are obtained for the flow parameters $R = 60$ and $\beta = 0.4$.

Fig. 3. The evolution of integral amplitudes of spatial harmonics of longitudinal velocity B_j in a reverse jet at sequential decrease of λ with 0.01 step starting from the critical point at $b = 0.787$; $\gamma_1 = 1.0276$, $\gamma_2 = 1.4746$, and $\gamma_3 = 4.0153$ are supercriticalities at the onset of different generation regimes, and $\lambda_a = 0.2129$ and $\lambda_b = 0.09286$ are the values of λ at the reference points.

$\hat{u}_j = -ik_j \hat{\psi}_j(y, t)$ [14]. This evolution diagram is obtained at $b = 0.787$. The coefficient λ decreases by 0.01 step starting from the critical value $\lambda_c \approx 0.3729$, at which the linear theory gives the phase velocity $c \approx 7.0 \cdot 10^{-5}$ (see curves 1 and 1' in Fig. 2). The numbers of spatial harmonics ($j = 7, 14, 6, \dots$) and the values of supercriticalities at the onset of different generation regimes (γ_1, γ_2 , and γ_3) are displayed in Fig. 3. Besides, two reference points $\lambda = \lambda_a = 0.2129$ and $\lambda = \lambda_b = 0.09286$ corresponding to the supercriticalities $\gamma_a = \lambda_c/\lambda_a \approx 1.752$ and $\gamma_b = \lambda_c/\lambda_b \approx 4.016$ are shown. The black stripes with irregular boundaries map the regime of dynamical chaos (see below for details). Their origin is associated with chaotic temporal modulation of B_j , the period of which is much less than the time intervals with constant λ .

Figure 3 shows that a stationary nonlinear wave with the wavenumber $k_7 = k_c \approx 1.47$ appears at $\gamma_1 \geq 1.0276$ (B_7 and B_{14} are the basic and the second spatial harmonic amplitudes, respectively). When $\gamma \geq \gamma_2 \approx 1.475$, the spatial harmonics B_6 , B_1 , and B_5 were born in the flow. The construction of the frequency-wavenumber spectrum of $\tilde{u}(x, y, t)$ at the level $y = 0$ described below showed that the increase of B_6 is caused by the excitation of a new mode with wave number k_6 and phase velocity $c \approx -0.18$ (in contrast to the phase velocity of the basic wave $c \approx -0.022$). The generation of the weaker harmonics B_1 and B_5 in Fig. 3 is due to nonlinear interactions between B_7 and B_6 (the processes $k_7 - k_6 = k_1$ and $k_6 - k_1 = k_5$). These “supplementary” (with respect to the initial chain of structures) perturbations create a possibility for the onset of anomalous transport. Finally, at $\gamma > \gamma_3 \approx 4.053$, dynamical chaos arises rapidly. Due to limitations of the numerical scheme, we restrict ourselves to studying the generation of an unsteady chain of structures at the initial stage of the transition to turbulence, when anomalous transport can occur at lower supercriticalities (the developed turbulence is not considered).

The computations showed that, with increasing supercriticality in Fig. 3, the phase velocity of the basic wave (drift velocity of the structure chain) slightly decreased. So, at $\gamma = \gamma_a$ and $\gamma = \gamma_b$ the values $c_a = -0.03$ and $c_b = -0.07$ were obtained. To find the solutions with an immovable chain, the reverse parameter b should be changed taking the initial conditions from the solution after the previous step in λ . Using the movie made of the computations of streamlines, it was revealed that at the reference points $\gamma = \gamma_a$ and $\gamma = \gamma_b$ the chains become immovable at $b = 0.775$ and $b = 0.747$, respectively. The parameters of the neutral waves found at these b are depicted in Fig. 2 by curves 2, 2' and 3, 3', where the critical values are $\lambda_c = 0.3523$, $k_c = 1.46$ and $\lambda_c = 0.3121$, $k_c = 1.435$, respectively. As follows from Fig. 2, the phase velocities at these critical points differ markedly from zero. The computations showed that the evolutionary diagrams drawn at $b = 0.775$ and $b = 0.747$ with the same $k_7 = 1.47$ are similar to the picture in Fig. 3, and the regimes with zero velocity of the chain were reached at the reference values $\lambda = \lambda_a$ and $\lambda = \lambda_b$. Due to lower critical values λ_c , the transition to these regimes (starting from the mentioned-above critical points in λ) occurs at lower supercriticalities $\gamma'_a \approx 1.655 < \gamma_a$ and $\gamma'_b \approx 3.361 < \gamma_b$. Thus, for the passage to different regimes with immovable chains of structures using step by step alteration of λ , it is necessary to find an appropriate value of b every time and to decrease λ starting from the critical point with nonzero phase velocity corresponding to this value of b .

The frequency-wavenumber spectra of the perturbations of the longitudinal velocity $\tilde{u}(x, y, t)$ obtained at the $y = 0$ level for the multiharmonic and chaotic regimes with immovable basic chains of wave structures designated by $A_{\omega k}$ are shown in Fig. 4. According to (3.1), the spectrum is discrete in k and quasi-continuous with respect to ω . The straight lines in the (ω, k) -plane correspond to zero phase velocities of the spatio-temporal harmonics. The spectrum of the chaotic regime presented in Fig. 4b is characterized by pedestals of the continuous spectrum in the vicinity of discrete peaks.

The occurrence of the dynamical chaos was confirmed by the computation of the largest Lyapunov exponent p immediately from the dynamical system generated by the numerical scheme (see analogously [11]). In particular, it was found that $p = 0.0155$ at $b = 0.747$, $\lambda = \lambda_b$. Note that the frequency-wavenumber spectra in the regimes with nonzero velocity of the basic wave marked by the points λ_a and λ_b in Fig. 3 insignificantly differ from the ones given in Fig. 4 (mainly, by a small phase velocity of the basic wave). A snapshot of streamlines obtained for the chaotic regime with an immovable chain of structures lying near the level $y = 1.45$ is depicted in Fig. 5. Note that the movie with this chain demonstrates disordered oscillations of closed streamlines in time. The fast flow mode available in Fig. 4 gives the separatrix contours

Fig. 4. The frequency-wavenumber spectra of the longitudinal velocity at the $y = 0$ level in the multi-harmonic (a) and chaotic (b) regimes of generation with an immovable chain of structures. Pictures (a) and (b) correspond to $b = 0.775$, $\lambda = \lambda_a$ and $b = 0.747$, $\lambda = \lambda_b$. The straight line in the (ω, k) -plane corresponds to the zero phase velocity of the harmonics.

Fig. 5. Snapshot of the streamlines of a reverse jet flow containing an immovable chain of structures near the $y = 1.6$ level in the fixed reference frame in the regime of dynamical chaos ($b = 0.747$, $\lambda = \lambda_b$).

located near the level $y = -1.2$, which are also displayed in Fig. 5. The comparison of the separatrix contours drawn in the reference frame co-moving with each chain of structures separately showed that their intersection is absent, i.e., the well-known Chirikov criterion for the transition to Lagrangian chaos is not met (a sample of the flow for which this criterion is confirmed was presented in [8]). However, in this case the Melnikov criterion may be used, which allows the thickness of the stochastic layers near the separatrix contours to be evaluated [5]. It should be added that the third chain in Fig. 5 appears intermittently near the level $y = -3.5$ where the mean velocity shift is fairly small.

In addition to the evolution diagrams with continuously decreasing λ at fixed β and R , the evolution of the regimes arising at a continuous increase of the flow velocity was studied, as it was done in [10, 11]. In this case, the supercriticality was defined as $\gamma = U/U_c$ (U_c is the critical velocity) and, with growing γ , all other parameters were altered since $\gamma = R/R_c = \lambda_c/\lambda = \beta_c/\beta$ (β_c is the value of β at $U = U_c$). The computation for the flow with $b = 0.787$ gave an evolution diagram similar to that in Fig. 3 including the onset of multiharmonic regime at $\gamma > \gamma_2 \approx 1.36$ and the appearance of dynamical chaos at $\gamma > \gamma_3 \approx 5.11$. A reason for the similarity is that the influence of β on the neutral curves in Fig. 2 is rather small.

To elucidate the role of the initial conditions with a decrease of λ , computations were made in which the initial problem with small initial perturbations on the background of flow with equilibrium velocity profile $u_0(y)$ was solved again after each step in λ . The pseudorandom sequence of harmonic phases entering small initial perturbations (noise seeding) was the same after each step. An essential influence of the initial conditions on the evolution of spatial harmonics amplitudes B_j was revealed. So, besides the multiharmonic regime shown in Fig. 3, the generation of the regular temporal modulation of the basic harmonic B_7 occurred at some steps in λ . Unlike the spectrum $A_{\omega k}$ presented in Fig. 4a, the supplementary discrete peak in this regime had the same wavenumber k_7 as the basic wave. In other words, the regime with temporal modulation of B_7 is a kind of the multiharmonic one. As in Fig. 3, the stripe with disordered boundaries corresponding to the dynamical chaos was obtained. Evidently, the occurrence of the modulation regime can be explained by the property of multistability of the system.

5. Anomalous transport

In order to describe the anomalous transport of a passive scalar by chains with zero drift velocity, the system (2.1)–(2.3) was supplemented by the equation for tracer particles [5]

$$\frac{dx_j}{dt} = \bar{u}(y_j, t) + \frac{\partial \tilde{\psi}}{\partial y_j}, \quad \frac{dy_j}{dt} = -\frac{\partial \tilde{\psi}}{\partial x_j}, \quad (5.1)$$

where j is the number of the tracer. The statistical characteristics of the tracer displacement $\delta x = x(t) - x(0)$ were computed for a basic (immovable) chain of structures. The initial conditions were specified in a thin layer 0.2 thick placed on either sides of the “pseudoseparatrix” contours passing near the boundaries of all closed streamlines. These contours were approximated by the eight-harmonics sections of the Fourier series with period π/k_1 (see analogously [11]).

Figure 6 shows an example of the tracer trajectories in the (x, t) -plane in the chaotic regime, the spectrum of which is presented in Fig. 6. They unambiguously illustrate spontaneous alternations of long flights of tracers (slanting sections) and their oscillations inside the closed streamlines (horizontal sections). The computations were made for two basic regimes of generation with immovable chains of structures described above, namely: $b = 0.775$, $\lambda = \lambda_a$ and $b = 0.747$, $\lambda = \lambda_b$. Using an ensemble of 1600 particles, the power dependences for mean particle displacement $M = \langle \delta x \rangle \sim t^s$ and variance $\sigma^2 = \langle (\delta x - \langle \delta x \rangle)^2 \rangle \sim t^\alpha$ (the brackets designate ensemble averaging) on time were obtained, which are characteristic for the anomalous transport [5, 6]. Establishment of the power laws and calculation of their exponents s and α for the multiharmonic and chaotic regimes with immovable chains of structures are illustrated by time dependences of $|M|$ and σ^2 given in Fig. 7.

As $\alpha > 1$ for both regimes, the anomalous diffusion of the passive scalar, which can be classified as superdiffusion, takes place [5] (at normal diffusion $\alpha = 1$). The results of computations of the power exponent s show that the advection in the multiharmonic regime is very close to the normal one (at which $s = 1$ rigorously). An exception is the advection in the chaotic regime presented by curve 2 in Fig. 7a, which does not show power dependence of the mean displacement on time. Within the kinematic model of anomalous transport, similar behavior of $|M|$ was noted in [5]. Thus, the performed analysis showed that the immovable unsteady chains of structures producing the anomalous diffusion of a passive scalar are possible in a jet flow with a complicated reverse velocity profile.

Fig. 6. Examples of the trajectories of the tracer particles with initial conditions in the vicinity of the boundaries of the immovable chain of structures in the regime of dynamical chaos ($b = 0.747$, $\lambda = \lambda_b$).

Fig. 7. Time dependence of the mean tracer displacement $|M|$ (a) and its dispersion σ^2 (b) in the multiharmonic (solid curves 1) and chaotic (solid curves 2) regimes of generation with immovable chains of structures and their power approximations (dashed lines). (a): 1 — $s = 0.994$, 2 — s is undefined, (b): 1 — $\alpha = 1.78$, 2 — $\alpha = 1.91$.

References

- [1] Vasavada, A.R. and Showman, A.P., Jovian Atmospheric Dynamics: An Update after Galileo and Cassini, *Rep. Prog. Phys.*, 2005, vol. 68, no. 8, pp. 1935–1996.
- [2] Nezlin, M.V. and Snezhkin, E.N., *Rossby Vortices, Spiral Structures, Solitons: Astrophysics and Plasma Physics in Shallow Water Experiments*, Berlin: Springer, 1993.
- [3] Haines, K.H. and Malanotte-Rizzoli, P., Isolated Anomalies in Westerly Jet Streams: A Unified Approach, *J. Atmos. Sci.*, 1991, vol. 48, no. 4, pp. 510–526.
- [4] Chkhetiani, O.G., Kalashnik, M.V., and Chagelishvili, G.D., Dynamics and Blocking of Rossby Waves in Quasi-Two-Dimensional Shear Flows, *JETP Lett.*, 2015, vol. 101, no. 2, pp. 79–84; see also: *Pis'ma v Zh. Èksp. Teoret. Fiz.*, 2015, vol. 101, no. 2, pp. 84–89.
- [5] del-Castillo-Negrete, D., Asymmetric Transport and Non-Gaussian Statistics of Passive Scalars

- in Vortices in Shear, *Phys. Fluids*, 1998, vol. 10, no. 3, pp. 576–594.
- [6] Koshel, K. V. and Prants, S. V., Chaotic Advection in the Ocean, *Physics-Uspekhi*, 2006, vol. 49, no. 11, pp. 1151–1178; see also: *Uspekhi Fiz. Nauk*, 2006, vol. 176, no. 11, pp. 1177–1206.
- [7] Wiggins, S., The Dynamical Systems Approach to Lagrangian Transport in Oceanic Flows, *Annu. Rev. Fluid Mech.*, 2005, vol. 37, pp. 295–328.
- [8] Finn, J. M. and del-Castillo-Negrete, D., Lagrangian Chaos and Eulerian Chaos in Shear Flow Dynamics, *Chaos*, 2001, vol. 11, no. 4, pp. 816–832.
- [9] Ryzhov, E. A. and Koshel, K. V., Resonance Phenomena in a Two-Layer Two-Vortex Shear Flow, *Chaos*, 2016, vol. 26, no. 11, 113116, 12 pp.
- [10] Shagalov, S. V., Reutov, V. P., and Rybushkina, G. V., Asymptotic Analysis of Transition to Turbulence and Chaotic Advection in Shear Zonal Flows on a Beta-Plane, *Izv. Atmos. Ocean. Phys.*, 2010, vol. 46, no. 1, pp. 95–108; see also: *Izv. AN. Fiz. Atmos. i Okeana*, 2010, vol. 46, no. 1, pp. 105–118.
- [11] Reutov, V. P. and Rybushkina, G. V., Anomalous Transport of a Passive Scalar at the Transition to Dynamical Chaos in a Barotropic Shear Layer, *Eur. J. Mech. B Fluids*, 2019, vol. 74, pp. 211–218.
- [12] Reutov, V. P. and Rybushkina, G. V., Unsteady Chains of Wave Structures and Anomalous Transport of a Passive Scalar in a Barotropic Jet Flow, *Izv. Atmos. Ocean. Phys.*, 2019, vol. 55, no. 6.
- [13] Dolzhanskii, F. V., Krymov, V. A., and Manin, D. Yu., Stability and Vortex Structures of Quasi-Two-Dimensional Shear Flows, *Sov. Phys. Usp.*, 1990, vol. 33, no. 7, pp. 495–520; see also: *Uspekhi Fiz. Nauk*, 1990, vol. 160, no. 7, pp. 1–47.
- [14] Reutov, V. P. and Rybushkina, G. V., Mode Switching and Dynamical Chaos in Quasi-Two-Dimensional Jet Flows, *Nelin. Mir*, 2016, vol. 14, no. 6, pp. 22–31 (Russian).
- [15] Orszag, S. A., Numerical Simulation of Incompressible Flows within Simple Boundaries: 1. Galerkin (Spectral) Representations, *Stud. Appl. Math.*, 1971, vol. 50, no. 4, pp. 293–327.
- [16] Reutov, V. P. and Rybushkina, G. V., Different-Scale Convective Structures in a Cooled Liquid Layer with a Horizontal Shear Flow, *Phys. Fluids*, 2013, vol. 25, no. 7, 074101, 16 pp.