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An Approach to Moving over Obstacles for a Wheeled Jumping Robot

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This paper approaches the issue of the jumping robot overcoming one step of a flight of stairs. A classification of obstacles according to the way they are surmounted is proposed, the basic concepts concerning the flight of stairs and the realization of a leap from one step to another are introduced. A numerical simulation of a robot's jump, carried out with a certain initial velocity, the vector of which is located at a certain angle to the horizon, has been carried out. The influence on the ranges of these two values of the numerical values of the height and the length of the step, the ratios between the length and the height of the step, as well as the distance to the step from which the jump is performed have been established.

Keywords: jumping robot, obstacle, flight of stairs, separation velocity, step dimensions

1. Introduction

Jumping robots feature a class of robots moving with a periodic separation from the surface [1–3]. Their advantage in comparison with other classes of robotic devices is a high maneuverability when moving on a rough terrain, as well as the ability to overcome various obstacles (fences, stairways, trenches, etc.) [4–6].

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Fig. 1. Obstacle classification.

Let all obstacles, when viewed in the vertical plane, be characterized by height h and length l . The robot also has height h_r and length l_r , and the robot's jump is characterized by height H and length L . Obstacles can be divided into surmountable and insurmountable ones (Fig. 1).

Insurmountable obstacles include those whose height is greater than the maximum possible height of the robot's jump $H_{\max}$:

$$h > H_{\max}. \quad (1.1)$$

The remaining obstacles will be considered surmountable; among them three classes are singled out. The first class ($K = 1$) includes narrow obstacles whose length is less than the length of the robot (Fig. 2a). The second class ($K = 2$) encompasses obstacles with a small length, the length of which is more than the length of the robot, but to overcome which two jumps are enough (during the first, jumping onto the obstacle occurs, and during the second, jumping off takes place), as shown in Fig. 2b.

Fig. 2. Ways of overcoming obstacles: a) $K = 1$, b) $K = 2$, c) $K = 3$.

The third class ($K = 3$) features obstacles with a large length. Overcoming them requires a series of jumps directly on the obstacle (Fig. 2c). Each of the options to surmount the obstacles can be characterized in accordance with the following formula:

$$K = \begin{cases} 1 & \text{if } (h < H_{\max}) \wedge (l < l_r) \wedge (k = 1), \\ 2 & \text{if } (h < H_{\max}) \wedge (l > l_r) \wedge (l < 2L) \wedge (k = 2), \\ 3 & \text{if } (h < H_{\max}) \wedge (l > l_r) \wedge (l \geq 2L) \wedge (k > 2) \end{cases} \quad (1.2)$$

where k stands for the number of jumps when obstacles are overcome.

It should be noted that in the specified classification the obstacles are conditionally considered in the form of rectangles. In the general case, their shape may be different: curved surfaces, multilevel (cascading) obstacles, alternation of straight sections with curvilinear, etc. In this paper, the focus is on the problem of a jumping robot overcoming the flight of stairs, which relates to cascade obstacles and is overcome by the $K = 3$ method.

2. Description of the task of overcoming the flight of stairs

We will consider the motion of a jumping robot up the flight of stairs, the number of steps in which is equal to n (Fig. 3). The computational scheme of the robot and the principles of its implementation are described in [7].

Fig. 3. Diagram of overcoming the flight of stairs by the jumping robot.

Let the length and the height of each step be equal to l and h , respectively. We assume that the dimensions of the robot are much smaller than those of the steps, which allows them to be neglected and to consider the robot as a material point of mass m . The device for making the jump acquires the velocity v_C , which is the velocity of separation of the device from the surface, the velocity vector is directed at the angle θ_C to the horizon. Before jumping, the robot is located at a distance x from the step.

$$x \in [l^*, l], \quad (2.1)$$

where $l^* = h \operatorname{ctg}(\theta_C^{\min})$ defines the distance within which the robot should not be located on the step due to the impossibility of making a jump from there while being limited by the height h of the step and the minimum angle of inclination of the velocity of separation $\theta_C^{\min}$. The characteristics of the robot's jump will include its length L and height H , calculated by the formulas:

$$L = v_C^2 \sin 2\theta_C / g, \quad L \in [x + (n-1)l, x + nl - l^*], \quad (2.2)$$

$$H = v_C^2 \sin^2 \theta_C / 2g, \quad H \in [nh, H_{\max}], \quad (2.3)$$

where g stands for the free-fall acceleration, and the distance $H_{\max}$ is limited by the height of the staircase ceiling. Overcoming the flight of stairs can be implemented using a different number of jumps p from 1 to the amount equaling the number of steps:

$$p \in [p_{\min}, p_{\max}], \quad p_{\min} = 1, \quad p_{\max} = n. \quad (2.4)$$

The number of steps overleaped in one jump is

$$N \in [N_{\min}, N_{\max}], \quad N_{\min} = 1 \text{ at } p_{\max}, \quad N_{\max} = n \text{ at } p_{\min}. \quad (2.5)$$

The distribution of surmountable steps over jumps can be described as follows:

$$\begin{aligned} N_1 &= (a_{\min} + (k - 1)), \quad k \in [1, k_1^*], \\ &\dots \\ N_i &= (a_{\min} + (k - 1)), \quad k \in [N_{i-1}, k_i^*] \text{ at } N_{i-1} \in [N_{(i-1)\min}, N_{(i-1)\max}], \\ &\dots \\ N_p &= n - \sum N_1 \dots N_{p-1} \text{ at } N_p \geq N_{p-1}, \end{aligned} \quad (2.6)$$

where $a_{\min} = 1$ denotes the minimum number of steps to be surmounted,

$$k_1^* = \begin{cases} \min(N_p), & \sum n_1 \dots n_p \bmod (p) = 0, \\ \min(N_p - 1), & \sum n_1 \dots n_p \bmod (p) \neq 0, \end{cases} \quad (2.7)$$

$$k_i^* = \begin{cases} \min(N_p), & \sum n_i \dots n_p \bmod (p - i + 1) = 0, \\ \min(N_p - 1), & \sum n_i \dots n_p \bmod (p - i + 1) \neq 0, \end{cases} \quad (2.8)$$

$\min(N_p)$ denotes the smallest number of steps to be overcome during the last jump.

According to the given formula, the number N_1 of the steps jumped over the first leap can vary from 1 to $\min(N_p)$ or $\min(N_p - 1)$ depending on whether the specified condition is fulfilled. The number N_i of the steps overleaped in jump i can vary from 1 to $\min(N_p)$ or $\min(N_p - 1)$ at each value N_{i-1} of the steps overleaped in the previous jump. The number N_p of the steps jumped over in the last jump is always represented as the difference between the number of steps in a flight of stairs and the number of steps already overleaped in the previous jumps. And this number cannot be less than the steps jumped over in the previous jump.

Let us consider the application of this formula for the case of $n = 4$ steps (Fig. 4).

The number of jumps can be equal to $p \in [1, 4]$:

- 1) at $p = 1$ the number of the steps to be leaped over $N_1 = 4$ (Fig. 4a),
- 2) at $p = 2$ the number of the steps to be leaped over $N_1/N_2 = 1/3, 2/2$ (Figs. 4b, 4c),
- 3) at $p = 3$ the number of the steps to be leaped over $N_1/N_2/N_3 = 1/1/2$ (Fig. 4d),
- 4) at $p = 4$ the number of the steps to be leaped over $N_1/N_2/N_3/N_4 = 1/1/1/1$ (Fig. 4e).

While determining the optimal way of overcoming the staircase span, the problem of minimizing the kinetic energy W of the robot at the moment of separation from the surface can be solved:

$$\min(W) = \min(mv_C^2/2). \quad (2.9)$$

Fig. 4. Options of overcoming the flight of stairs: a) $p = 1$, b, c) $p = 2$, d) $p = 3$, e) $p = 4$.

Next, we are going to focus our attention on the robot's jumping onto one step of a flight of stairs. The purpose of the study is to determine the allowable ranges of the separation velocity of the robot from the surface and the angle of inclination of the vector of this velocity to the horizontal, and to establish the impact of the geometric dimensions of the step on these ranges and the distance to the step from which the jump occurs. As a result of the work, recommendations on the implementation of the jump with the minimum energy consumed to perform it will be formulated.

3. Ranges of the surface separation angle of the robot

We will conduct a numerical study of the robot's jump in dimensionless quantities (the angles are given in degrees) when the device jumps onto a step whose length and height are related by

$$s = l/h. \quad (3.1)$$

First, we define the ranges of permissible values of the angle $\theta_C^{\min}$ and the corresponding values of $x_{\min}$, as well as the values of $x_{\max}$, due to the length of the step. In view of the formula (2.1), we can write

$$\theta_C^{\min} = \arctg(h/x_{\min}), \quad (3.2)$$

$$x_{\max} = l. \quad (3.3)$$

Figure 5 shows the dependences of the maximum and minimum values of x on the smallest angle of inclination of the velocity vector of separation of the robot from the surface with three ratios between the length and the height of the step.

From the above graphs one can see that the range of permissible values of x and the angles $\theta_C^{\min}$ (colored blue) are limited above by the value $x_{\max}$, and below by the curve $x_{\min}$, while, with increasing ratio s , the specified area expands, which is more clearly illustrated by the following graph (Fig. 6a).

The dependences in Fig. 6a are constructed for the case when the robot's jump occurs from the starting point of the step, i.e., $x = x_{\max}$. We will restrict ourselves to the range of possible separation angles of the robot from the surface $\theta_C = [30^\circ, 80^\circ]$, this corresponds to the range $s = [0.2, 1.8]$. The analysis of dependences allows the conclusion that the longer the step with respect to its height, the smaller the angle of inclination of the velocity vector to the

Fig. 5. Dependence graphs of $x_{\max}(\theta_C^{\min})$, $x_{\min}(\theta_C^{\min})$ at $h = 0.2$: a) $s = 0.6$, b) $s = 1$, c) $s = 1.4$.

Fig. 6. Dependence graphs of $\theta_C^{\min}(s)$, $\theta_C^{\max}(s)$, drawn with the consideration of: a) the formula (2.1) at $x = x_{\max}$, b, c) robot jump simulation, b) at $x = x_{\max}$, c) at $x = 0.75x_{\max}$.

horizon at which the robot is capable of jumping onto the step. It should be noted that the type of graph in Fig. 6a and its numerical values do not depend on the values of the length and height of the step, but are determined only by the ratio s . The graphs shown in Fig. 6b and Fig. 6c will be explained in further sections of the paper.

4. Trajectories of the robot's jump onto the step

Figure 7 shows graphs of the trajectories of the robot's motion with the equality of the length and the height of the step ($s = 1$) at three values of the angle θ_C from those defined by Fig. 6a range of values $\theta_C = [45^\circ, 80^\circ]$, at which the jump onto the step is possible.

The graphs show that with the specified range of the angles θ_C it is possible to implement such a robot jump that it overcomes one step (rather than hitting its side surface, without reaching it). But at the same time, at the angle $\theta_C = 45^\circ$ (Fig. 7a) the robot cannot land on the necessary (first) step of the flight of stairs, since the speed acquired by him at separation is such that the device flies over the first step and hits the side surface of the second step. Its further behavior will be subject to the impact properties of the surface (in Fig. 7a it is not shown). Such a landing option is unacceptable.

To analyze the graphs presented in Figs. 7b, 7c we introduce the concept of the minimum $v_C^{\min}$ and maximum $v_C^{\max}$ velocity values. By $v_C^{\min}$ we mean the minimum possible speed of the robot's separation from the surface, which ensures the landing of the robot onto the step. The length of the jump is the minimum possible, in extreme cases it is equal to $L_{\min}$ (when

Fig. 7. Dependence graphs of $y_C(x_C)$ at $s = 1$, $h = 0.2$: a) $\theta_C = 45^\circ$, b) $\theta_C = 60^\circ$, c) $\theta_C = 80^\circ$, 1 — $v_C = v_C^{\min}$, 2 — $v_C = v_C^{\max}$.

landing onto the starting point of the step) and $L_{\max}$ (when landing onto the end point of the step), otherwise it lies within the range $(L_{\min}, L_{\max})$. By $v_C^{\max}$ we mean the maximum possible separation velocity of the robot from the surface, at which the landing at the end point of the step occurs, which is described by the formula

$$v_C = \begin{cases} v_C^{\min} & \text{if } H = h, L = \begin{cases} L_{\min} & \text{if } x_{Ck} = l, y_{Ck} = h, \\ \in (L_{\min}, L_{\max}) & \text{if } l < x_{Ck} < 2l, y_{Ck} = h, \\ L_{\max} & \text{if } x_{Ck} = 2l, y_{Ck} = h, \end{cases} \\ v_C^{\max} & \text{if } H = h, L = L_{\max}. \end{cases} \quad (4.1)$$

Figure 7b demonstrates a jump at $\theta_C = 60^\circ$, in this case, with the minimum allowable separation velocity $v_C^{\min}$ (curve 1) the robot lands onto the first step, but not on its extreme point, as is the case, for example, at a larger angle value $\theta_C = 80^\circ$ (Fig. 7c), but onto some point of the step. Also, Figs. 7b, 7c show the trajectories of the robot at separation with the maximum possible velocity to jump onto the step $v_C^{\max}$ (curve 2).

The analysis of the graphs constructed in Fig. 7 shows that the minimum angle of inclination of the velocity vector of the robot's separation from the surface cannot be determined only from the geometric ratios (relations) given in the previous section. Its value requires an adjustment determined by the behavior of the robot in flight, i.e., it can be established as a result of the numerical simulation, which is illustrated in Fig. 6b. According to the corrected graphs, it can be seen that the range of ratios of the length and the height of the step s equals $s \in [0.26, 2.6]$, the minimum and the maximum values of s is more than the corresponding values of the graph in Fig. 6a, this means that the range of s shifts and expands in the direction of increasing the s values.

5. Ranges of the separation velocities of the robot when jumping

In the previous sections, the ranges of permissible values of the angle of inclination of the robot's separation velocity vector from the surface for jumping onto the step were determined. Here we turn our attention to the ranges of separation velocities allowing the robot to make the specified jump. First, we consider how the ratio s of the length and the height of the step affects the values $v_C^{\min}$ and $v_C^{\max}$. Figure 8 shows the dependence graphs of the minimum and

Fig. 8. Dependence graphs of $v_C^{\max}(\theta_C)$, $v_C^{\min}(\theta_C)$ at $h = 0.2$, $x = x_{\max}$: a) $s = 0.8$, b) $s = 1$, c) $s = 1.4$.

maximum values of the robot's separation velocity from the surface, providing the jump onto the step, for three ratios of the length and the height of the latter.

The graphs show that the range of permissible values of the angle θ_C expands as the parameter s increases. Between the constructed curves, there is a range of possible values of the object separation velocity, and two areas are marked in it, denoted by numbers 1 and 2. Area 1 corresponds to the case of the robot landing not onto the starting point of the step (as shown in Fig. 7b), and area 2 corresponds to the case of landing of the device at the starting point of the step (as illustrated in Fig. 7c). The values $v_C^{\min}$ and $v_C^{\max}$, corresponding to the same value of θ_C , increase with increasing parameter s , i.e., the ranges of the separation velocity of the robot from the surface are expanding.

Let us consider how the ranges of permissible velocities are influenced by the numerical values of the length and the height of the step at the constant value s (Fig. 9).

Fig. 9. Dependence graphs of $v_C^{\max}(\theta_C)$, $v_C^{\min}(\theta_C)$ at $s = 1$, $x = x_{\max}$: a) $h = 0.2$, b) $h = 0.4$, c) $h = 0.6$.

From the graphs given above it can be seen that the range of permissible angles θ_C does not depend on the numerical values of the dimensions of the step, but is determined only by the ratio of its height and length (parameter s). The ranges of the separation velocity of the robot from the surface expand as the dimensions of the step increase, and there is also a "rise" of the entire range.

Another parameter affecting the ranges of the separation velocity is the distance x to the step from which the jump occurs. The graphs in Fig. 8 and Fig. 9 are built for the cases when $x = x_{\max}$, and in Fig. 10 the same dependencies are also obtained for the options $x = 0.5x_{\max}$ and $x = 0.75x_{\max}$.

Fig. 10. Dependence graphs of $v_C^{\max}(\theta_C)$, $v_C^{\min}(\theta_C)$ at $s = 1$, $h = 0.2$: a) $x = 0.5x_{\max}$, b) $x = 0.75x_{\max}$, c) $x = x_{\max}$.

From the graphs obtained, it can be seen that as the value of x decreases, the range of allowable angles θ_C is observed to narrow. Thus, this dependence is nonlinear. Also, there is an increase in area 1 and a reduction in area 2, which characterize the landing variant of the robot (at $x = x_{\max}$ area 1 comprises $1/3$ from the whole range of the angles θ_C , and at $x = 0.5x_{\max}$ $1/2$ from the range of the angles θ_C). In addition, it should be noted that the smallest velocity value $v_C^{\min}$ is observed when jumping from the smallest of the considered distances to the step ($x = 0.5x_{\max}$) at the angle θ_C , corresponding to the boundary of areas 1 and 2 (Fig. 10a).

For more visual clarity, the effect of the distance x impact on the ranges s and $\theta_C^{\min}$ can be traced while analyzing Fig. 6b and Fig. 6c. According to the graphs presented there, it is obvious that to change $\theta_C^{\min}$ within the range $\theta_C^{\min} \in [30^\circ, 80^\circ]$ at $x = 0.75x_{\max}$, a wider range of s values is required, as well as its shift towards the increase of s compared to the case when $x = x_{\max}$.

6. Conclusion

The present paper is devoted to the issues of overcoming an obstacle in the form of a flight of stairs by a jumping robot. Based on the classification introduced in the paper, it was established that the flight of stairs can be classified as an obstacle that requires overcoming by jumping onto them, making a series of jumps on the obstacle and jumping off the obstacle. The article introduces basic concepts related to the robot's jumping onto the step of a flight of stairs; a numerical simulation of the device movement has been carried out for the case when the step dimensions are significantly larger than the robot's dimensions, which allows for the simulation of the latter in the form of a material point.

As a result of the simulation, it has been established that the range of allowable angles of inclination of the velocity vector of the robot's separation from the surface, ensuring the device landing onto the step, is determined by the ratio of the height and the length of the step, as well as the distance to the step from which jumping occurs, but does not depend on the numerical values of the step dimensions.

In addition, diagrams have been constructed for the areas where the robot jumps onto the step are implemented: a jump onto the starting point of the stage is possible (area 2) or not possible (area 1) depending on the device separation velocity and the angle of inclination of this velocity vector to the horizontal. As a result of the analysis of these graphs, it has been established that area 1 is observed at the beginning of the range of allowable separation angles of the robot from the surface, and then transition to area 2 takes place.

It was also established that for the implementation of the jump with the lowest possible velocity, the ratio between the length and the height of the step, the numerical values of the height and the length of the step, and the distance to the step from which the jump is made must be the smallest possible.

The resulting regularities can be further used to develop the optimal way of overcoming the flight of stairs, for instance, when it comes to minimizing the energy expended on the movement.

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